STRUCTURE OF HADRONS IN HARD PROCESSES

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Abstract. In these lectures I want to discuss how the structure functions in deep inelastic scattering relate to quark and gluon correlation functions. In particular we will consider the issue of intrinsic transverse momenta of quarks, which becomes important in processes like 1-particle inclusive leptoproduction. Some examples of cross sections and asymmetries, in particular in polarized scattering processes are discussed.

1. Introduction

The central point of these lectures is the availability of a field theoretical framework for the strong interactions. It is the nonabelian gauge theory based on the color symmetry group $SU(N_c)$ with $N_c = 3$. This theory, Quantum Chromodynamics (QCD) underlies the strong interactions of which the existence is known since the thirties. At that time the goal was to understand the forces in the atomic nucleus. The experimental results of many experiments revealed the existence of a rich spectrum of hadrons, baryons and mesons. The presence of these excitation spectra and also explicit measurements of the charge and current distributions of the hadrons revealed the composite nature of hadrons. The quark model brought some order into this situation, describing baryons as composite systems of three valence quarks and mesons as a composite system of a valence quark and antiquark. Valence refers here to the fact that these quarks are the contributors to the quantum numbers (upness, downness, strangeness, etc.) of the hadrons. The symmetry considerations based on flavor $SU(3)$ and flavor-spin $SU(6)$ are the basis of the success of the quark model. Color was introduced in a rather early stage to solve a number of problems such as the fermion statistics of the quarks. The experimental results of

the SLAC-MIT deep inelastic electron scattering experiments, which indicated the existence of hard 'point-like' constituents in the nucleon and the field-theoretical developments in nonabelian gauge theories, specifically the notion of asymptotic freedom and the proof of renormalizability, led to the natural emergence of QCD as the theory for the strong interactions between the quarks and gluons. The important feature of QCD is the fact that the force becomes weaker at short distances. This anti-screening behavior or asymptotic freedom is a unique feature of non-abelian gauge theories.

QCD is part of the Standard Model that describes the strong and electroweak forces. All forces in the Standard Model are described within the framework of nonabelian gauge theories, based on a gauge symmetry. For testing directly QCD, one preferably avoids the presence of hadrons, as these constitute complex bound state systems. Useful scattering processes for this purpose are

$e^+e^- \longrightarrow X$	(inclusive annihilation),
$e^+e^- \longrightarrow \text{jets}$	(jet production),
$\ell H \longrightarrow \ell' X$	(inclusive leptoproduction).

Analyzing jets produced in e^+e^- scattering represent, modulo complications arising from hadronization, hard scattering processes $e^+e^- \to q\bar{q}$, $e^+e^- \to q\bar{q}g$, etc. For these processes perturbative QCD can be used to do reliable calculations. Similarly one can use perturbative QCD to study the $\ln Q^2$ dependence of the elementary $\ell q \to \ell' q$ cross sections that are important in leptoproduction.

The properties of hadrons are poorly described directly starting from QCD. They require a nonperturbative approach in quantum field theory. This has led to the use of models that incorporate some features like confinement and asymptotic freedom and symmetries of the underlying theory, such as quark potential models, bag models or chiral models. Most promising from a fundamental point of view are lattice gauge approaches.

Controlling and selectively probing the nonperturbative regime in high energy scattering processes is the key to study the structure of hadrons in the context of QCD. The control parameters for the target and the probe are the spin and flavor, which in combination with the kinematical flexibility in scattering processes is used to select the observable and its gluonic or quarkic nature. Examples are

$\ell H \longrightarrow \ell' H$	(elastic leptoproduction)	(spacelike) form factors,
$\ell H \longrightarrow \ell' X$	(inclusive leptoproduction)	distribution functions,
$\ell H \longrightarrow \ell' h X$	(1-particle inclusive leptoproduction)	distribution and fragmentation functions,
$e^+e^- \longrightarrow h\bar{h}$	(annihilation into $h\bar{h}$)	(timelike) form factors,
$e^+e^- \longrightarrow hX$	(1-particle inclusive annihilation)	fragmentation functions,

$H_1 H_2 \longrightarrow \mu^+\mu^- X$ (Drell-Yan scattering) distribution functions.

These notes consist of two parts. After this introduction we discuss the basic formalism to deal with the beforementioned processes, introducing the hadron tensor, form factors, structure functions. These are needed to write down general expressions for the electroweak cross sections. The second part reviews the introduction of parton distribution and fragmentation functions that enable a systematic treatment of the structure functions at high energies and/or momentum transfer.

2. Inclusive Leptoproduction

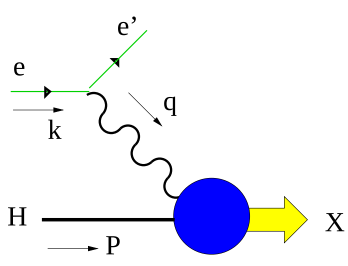
2.1. THE HADRON TENSOR

For the process $\ell{+}H \to \ell'{+}X$ (see Fig. 1), the cross section can be separated into a lepton and hadron part. Although the lepton part is simpler, let us start with the hadron part,

$$2M\,W^{(\ell H)}_{\mu\nu}(q;PS) = \frac{1}{2\pi}\sum_X \int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X) \times\langle PS|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|PS\rangle, \quad (1)$$

The simplest thing to do is to parametrize this tensor in standard tensors and structure functions. Instead of the traditional choice [1] for these tensors, $g_{\mu\nu}$ and $P_\mu P_\nu$ and structure functions W_1 and W_2, we immediately go to a dimensionless representation, using a natural space-like momentum (defined by q) and a time-like momentum constructed from P and q,

$$\hat{z}^\mu = -\hat{q}^\mu \quad = \quad -\frac{q^\mu}{Q}, \quad (2)$$



$$\ell H \longrightarrow \ell' X$$

$$x_{_B} = \frac{Q^2}{2P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot k}$$

Figure 1. Momenta and invariants in inclusive leptoproduction. The scale is set by the invariant momentum squared of the virtual photon, $q^2 \equiv -Q^2$, which for a hard process becomes $Q^2 \to \infty$.

$$\hat{t}^\mu = \frac{\tilde{P}^\mu}{\sqrt{\tilde{P}^2}} = \frac{1}{\sqrt{\tilde{P}^2}}\left(P^\mu - \frac{P\cdot q}{q^2}\,q^\mu\right) = \frac{q^\mu + 2x_{_B}\,P^\mu}{Q}. \quad (3)$$

Using hermiticity for the currents, parity invariance and current conservation one obtains as the most general form the symmetric tensor

$$M\,W_S^{\mu\nu}(q,P) = \underbrace{\left(-g^{\mu\nu} + \hat{q}^\mu\hat{q}^\nu - \hat{t}^\mu\hat{t}^\nu\right)}_{-g_\perp^{\mu\nu}} F_1 + \hat{t}^\mu\hat{t}^\nu\,\underbrace{\left(\frac{F_2}{2x_{_B}} - F_1\right)}_{F_L}, \quad (4)$$

where the structure functions F_1 and F_2 or the transverse and longitudinal structure functions, $F_T = F_1$ and F_L, depend only on the for the hadron part relevant invariants Q^2 and $x_{_B}$. In all equations given here we have omitted target mass effects of order M^2/Q^2.

2.2. THE LEPTON TENSOR

In order to write down the cross section one needs to include the necessary phase space factors and include the lepton part given by the tensor

$$L_{\mu\nu}^{(\ell H)}(k\lambda;k'\lambda') = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2 g_{\mu\nu} + 2i\lambda_e\,\epsilon_{\mu\nu\rho\sigma}q^\rho k^\sigma. \quad (5)$$

We have included here the (longitudinal) lepton polarization ($\lambda_e = \pm 1$). For later convenience it is useful to rewrite this tensor also in terms of the space-like and time-like vectors $\hat{q}$ and $\hat{t}$. It is a straightforward exercise to get

$$k^\mu = \frac{Q}{2}\,\hat{q}^\mu + \frac{(2-y)Q}{2y}\,\hat{t}^\mu + \frac{Q\sqrt{1-y}}{y}\,\hat{\ell}^\mu, \quad (6)$$

where $\hat{\ell}$ is the perpendicular direction defining the lepton scattering plane (see Fig. 2). This perpendicular direction becomes relevant only if other vectors than P and q are present, e.g. a spin direction in polarized scattering or the momentum of a produced hadron in 1-particle inclusive processes. The lepton tensor becomes

$$\begin{aligned} L_{(\ell H)}^{\mu\nu} = \frac{Q^2}{y^2}\Bigg[& -2\left(1-y+\frac{1}{2}\,y^2\right) g_\perp^{\mu\nu} + 4(1-y)\hat{t}^\mu\hat{t}^\nu \\ & +4(1-y)\left(\hat{\ell}^\mu\hat{\ell}^\nu + \frac{1}{2}\,g_\perp^{\mu\nu}\right) + 2(2-y)\sqrt{1-y}\;\hat{t}^{\{\mu}\hat{\ell}^{\nu\}} \\ & -i\lambda_e\,y(2-y)\,\epsilon_\perp^{\mu\nu} - 2i\lambda_e\,y\sqrt{1-y}\;\hat{\ell}_\rho\epsilon_\perp^{\rho\,[\mu}\hat{t}^{\nu]}\Bigg], \end{aligned} \quad (7)$$

where $\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma}\hat{t}_\rho\hat{q}_\sigma$.

2.3. THE INCLUSIVE CROSS SECTION

The cross section for unpolarized lepton and hadron only involves the first two (symmetric) terms in the lepton tensor and one obtains

$$\frac{d\sigma_{OO}}{dx_B\,dy} = \frac{4\pi\,\alpha^2\,x_B\,s}{Q^4}\Bigg\{\left(1-y+\frac{1}{2}\,y^2\right)F_T(x_B,Q^2) + (1-y)\,F_L(x_B,Q^2)\Bigg\}. \tag{8}$$

As soon as the exchange of a Z^0 boson becomes important the hadron tensor is no longer constrained by parity invariance and a third structure function F_3 becomes important.

2.4. TARGET POLARIZATION

The use of polarization in leptoproduction provides new ways to probe the hadron target. For a spin 1/2 particle the initial state is described by a 2-dimensional spin density matrix $\rho = \sum_\alpha |\alpha\rangle p_\alpha \langle\alpha|$ describing the probabilities p_α for a variety of spin possibilities. This density matrix is hermitean with $\mathrm{Tr}\,\rho = 1$. It can in the target rest frame be expanded

$$\rho_{ss'} = \frac{1}{2}\left(1 + \boldsymbol{S}\cdot\boldsymbol{\sigma}_{ss'}\right), \tag{9}$$

where $\boldsymbol{S}$ is the spin vector. When $|\boldsymbol{S}| = 1$ one has a pure state (only one state $|\alpha\rangle$ and $\rho^2 = \rho$), when $|\boldsymbol{S}| \le 1$ one has an ensemble of states. For the case $|\boldsymbol{S}| = 0$ one has simply an averaging over spins, corresponding to an unpolarized ensemble. To include spin one could generalize the hadron tensor to a matrix in spin space, $\tilde{W}^{\mu\nu}_{s's}(q,P) \propto \langle P,s'|J^\mu|X\rangle\langle X|J^\nu|P,s>$ depending only on the momenta or one can look at the tensor $\sum_\alpha p_\alpha \tilde{W}^{\mu\nu}_{\alpha\alpha}(q,P)$, which is given by

$$W^{\mu\nu}(q,P,S) = \mathrm{Tr}\left(\rho(P,S)\tilde{W}^{\mu\nu}(q,P)\right), \tag{10}$$

with the spacelike spin vector S appearing *linearly* and in an arbitrary frame satisfying $P\cdot S = 0$. It has invariant length $-1 \le S^2 \le 0$. It is convenient to write the spin vector as

$$S^\mu = \frac{\lambda}{M}\left(P^\mu - \frac{2x_B M^2}{Q^2}\,q^\mu\right) + S_\perp^\mu, \tag{11}$$

with $\lambda = M(S\cdot q)/(P\cdot q)$. For a pure state one has $\lambda^2 + \boldsymbol{S}_\perp^2 = 1$. Using symmetry constraints one obtains for electromagnetic interactions (parity

conservation) an antisymmetric part in the hadron tensor,

$$M\,W_A^{\mu\nu}(q,P,S) = \underbrace{-i\,\lambda\,\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma}{P\cdot q}}_{-i\,\lambda\,\epsilon_\perp^{\mu\nu}}\,g_1 + i\,\frac{2Mx_{_B}}{Q}\,\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,g_T. \qquad (12)$$

The polarized part of the cross section becomes

$$\frac{d\sigma_{LL}}{dx_{_B}dy} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\Bigg\{\lambda\,\left(1-\frac{y}{2}\right)g_1(x_{_B},Q^2) - |S_\perp|\,\cos\,\phi_S^\ell\,\frac{2Mx_{_B}}{Q}\sqrt{1-y}\;g_T(x_{_B},Q^2)\Bigg\}. \qquad (13)$$

3. Semi-inclusive leptoproduction

3.1. THE HADRON TENSOR

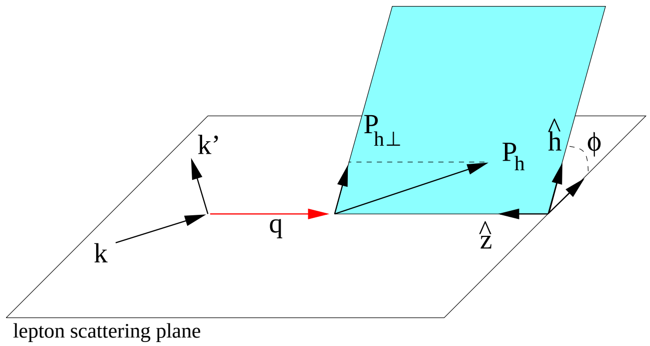
More flexibility in probing new aspects of hadron structure is achieved in semi-inclusive scattering processes. For instance in 1-particle inclusive measurements one can measure azimuthal dependences in the cross sections. The central object of interest for 1-particle inclusive leptoproduction, the hadron tensor, is given by

$$2M\mathcal{W}_{\mu\nu}^{(\ell H)}(q;PS;P_hS_h) = \sum_X\int\frac{d^3P_X}{(2\pi)^3 2P_X^0}\delta^4(q+P-P_X-P_h) \times\langle PS|J_\mu(0)|P_X;P_hS_h\rangle\langle P_X;P_hS_h|J_\nu(0)|PS\rangle, \qquad (14)$$

where P, S and P_h, S_h are the momenta and spin vectors of target hadron and produced hadron, q is the (space-like) momentum transfer with $-q^2$ $= Q^2$ sufficiently large. The kinematics is illustrated in Fig. 2, where also the scaling variables are introduced. For the parametrization of the hadron tensor in terms of structure functions it is useful to introduce the directions $\hat{q}$ and $\hat{t}$ as before and using the vector P_h to construct a vector that is orthogonal to these vectors. For the situation that $P\cdot P_h$ is $\mathcal{O}(Q^2)$ (current fragmentation!) one finds that

$$q_T^\mu = q^\mu + x_{_B}\,P^\mu - \frac{P_h^\mu}{z_h} = -\frac{P_{h\perp}^\mu}{z_h} \equiv -Q_T\,\hat{h}^\mu, \qquad (15)$$

is such a vector. This vector is proportional to the transverse momentum of the outgoing hadron with respect ot P and q. It can also be considered as the transverse momentum of the photon with respect to the hadron



$$\ell H \longrightarrow \ell' h X$$

$$x_{_B} = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot k}$$
$$z_h = \frac{P\cdot P_h}{P\cdot q}$$
$$Q_T = \frac{|P_{h\perp}|}{z_h}$$

Figure 2. Kinematics for 1-particle inclusive leptoproduction.

momenta P and P_h. For an unpolarized (or spin 0) hadron in the final state the symmetric part of the tensor is given by

$$\begin{aligned} M\mathcal{W}_S^{\mu\nu}(q,P,P_h) &= -g_\perp^{\mu\nu}\,\mathcal{H}_T + \hat{t}^\mu\hat{t}^\nu\,\mathcal{H}_L \\ &\quad + \hat{t}^{\,\{\mu}\hat{h}^{\nu\}}\,\mathcal{H}_{LT} + \left(2\,\hat{h}^\mu\hat{h}^\nu + g_\perp^{\mu\nu}\right)\mathcal{H}_{TT}. \end{aligned} \tag{16}$$

Noteworthy is that also an antisymmetric term in the tensor is allowed,

$$M\mathcal{W}_A^{\mu\nu}(q,P,P_h) = -i\hat{t}^{\,[\mu}\hat{h}^{\nu]}\,\mathcal{H}'_{LT}. \tag{17}$$

3.2. THE SEMI-INCLUSIVE CROSS SECTION

Clearly the lepton tensor in Eq. 7 is able to distinguish all the structures in the semi-inclusive hadron tensor. The symmetric part gives the cross section for unpolarized leptons,

$$\begin{aligned} \frac{d\sigma_{OO}}{dx_{_B}\,dy\,dz_h d^2q_T} &= \frac{4\pi\,\alpha^2\,s}{Q^4}\,x_{_B} z_h \Bigg\{\left(1-y+\frac{1}{2}\,y^2\right)\mathcal{H}_T + (1-y)\,\mathcal{H}_L \\ &\qquad - (2-y)\sqrt{1-y}\,\cos\phi_h^\ell\;\mathcal{H}_{LT} \\ &\qquad + (1-y)\,\cos 2\phi_h^\ell\;\mathcal{H}_{TT}\Bigg\} \end{aligned} \tag{18}$$

while the antisymmetric part gives the cross section for a polarized lepton (note the target is not polarized!)

$$\frac{d\sigma_{LO}}{dx_{_B}\,dy\,dz_h d^2q_T} = \lambda_e\,\frac{4\pi\,\alpha^2}{Q^2}\,z_h\,\sqrt{1-y}\,\sin\phi_h^\ell\;\mathcal{H}'_{LT}. \tag{19}$$

Of course many more structure functions appear for polarized targets or if one considers polarimetry in the final state.

4. Form factors

A special case is the situation in which the final state is identical to the initial state, elastic scattering. In that case the final state four momentum is $P' = P + q$ and is fixed to be $(P+q)^2 = M^2$, i.e. $x_B = 1$. We can still use the formalism for inclusive leptoproduction but the hadron tensor becomes becomes

$$2M\, W_{\mu\nu}(q,P) = \underbrace{\langle P\vert J_\mu(0)\vert P'\rangle\langle P'\vert J_\nu(0)\vert P\rangle}_{H_{\mu\nu}(P;P')}\,\frac{1}{Q^2}\,\delta(1-x_B). \tag{20}$$

One needs the current matrix elements of the electromagnetic current, which using hermiticity, parity and current conservation can be parametrized in terms of (real) form factors, for a spin 1/2 particle

$$\langle P',S'\vert J_\mu(x)\vert P,S\rangle = e^{i\,q\cdot x}\,\bar U_{S'}(P')\underbrace{\left[\gamma_\mu\,F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}\,F_2(Q^2)\right]}_{\Gamma_\mu(P,P')}U_S(P), \tag{21}$$

where $U_S(P)$ are the standard Dirac spinors.

In order to interpret the form factors for spacelike q, note that in leptoproduction there always exist a frame in which q is purely spacelike ($q^0 = 0$), the socalled brick-wall or Breit frame. Working out the current expression for the nucleon in this frame ($P'^0 = P^0$, $\boldsymbol{P} = -\boldsymbol{q}/2$, $\boldsymbol{P}' = +\boldsymbol{q}/2$), gives

$$\begin{aligned}
\langle P',S'\vert J_0^{em}\vert P,S\rangle &= 2M\left[F_1(Q^2) - \frac{Q^2}{4M^2}\,F_2(Q^2)\right]e^{iq\cdot x} \\
&\equiv 2M\,G_E(Q^2)e^{iq\cdot x}, \qquad (22)\\
\langle P',S'\vert \boldsymbol{J}^{em}\vert P,S\rangle &= \left[F_1(Q^2)+F_2(Q^2)\right]\,(i\boldsymbol{\sigma}_N\times\boldsymbol{q})e^{iq\cdot x} \\
&\equiv G_M(Q^2)\,(i\boldsymbol{\sigma}_N\times\boldsymbol{q})e^{iq\cdot x}, \qquad (23)
\end{aligned}$$

where $\chi^\dagger_{S'}\,\boldsymbol{\sigma}\,\chi_S \equiv \boldsymbol{\sigma}_N$. These expressions show the relevance of the Sachs form factors G_E and G_M being the Fourier transfer of the spatial charge and current distribution. The quantity $e\,G_E(0)$ is the charge of the nucleon, $e\,G_M(0)/M$ is the magnetic moment of the nucleon. The quantity $\kappa = F_2(0) = G_M(0) - G_E(0)$ is the anomalous magnetic moment.

One has for a point-particle (e.g. electron or muon) (in lowest order in α)

$$F_1(0) = G_E(0) = 1, \qquad F_2(0) = 0, \qquad G_M(0) = 1,$$

and no Q^2-dependence, while for a composite particle like the nucleon one has a combination of quark form factors weighted by the charges, giving

$$\begin{array}{lll} F_1^p(0) = G_E^p(0) = 1, & F_2^p(0) = \kappa_p \approx 1.79, & G_M^p = \mu_p \approx 2.79, \\ F_1^n(0) = G_E^n(0) = 0, & F_2^n(0) = \kappa_n \approx -1.91, & G_M^n = \mu_n \approx -1.91. \end{array}$$

The Q^2-dependence for the electromagnetic form factors of the nucleon approximately is given by

$$G_E^p(Q^2) \approx \frac{G_M^p(Q^2)}{\mu_p} \approx \frac{G_M^n(Q^2)}{\mu_n} \approx \frac{1}{\left(1+Q^2/0.69\ \mathrm{GeV}^2\right)^2}$$

(dipole form factors).

For the tensor $H_{\mu\nu}$ one obtains the result

$$\begin{aligned} H_{\mu\nu}(P,P') &= \frac{1}{2}\sum_{S,S'} \langle P,S|J_\mu(0)|P',S'\rangle\langle P',S'|J_\nu(0)|P,S\rangle \\ &= \frac{1}{2} Tr\ \left[\Gamma_\mu(\not{P}'+M)\Gamma_\nu(\not{P}+M)\right] \\ &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) Q^2(F_1+F_2)^2 + 4\,\tilde{P}_\mu\tilde{P}_\nu \left(F_1^2 + \frac{Q^2}{4M^2}\,F_2^2\right) \\ &= -g_{\perp\,\mu\nu}\,Q^2 G_M^2 + \hat{t}_\mu\hat{t}_\nu\,4M^2\,G_E^2 \end{aligned} \tag{24}$$

One thus sees the following *elastic* contribution in the structure functions

$$F_T(x_B,Q^2) = G_M^2(Q^2)\,\delta(1-x_B), \tag{25}$$

$$F_L(x_B,Q^2) = \frac{4M^2}{Q^2}\,G_E^2(Q^2)\,\delta(1-x_B). \tag{26}$$

In particular when one is also considering other currents than the electromagnetic case, it is useful to realize that the currents for the γ, Z- or W-particles of course all are known in terms of quark vector and axial vector currents, e.g.

$$J_\mu^{(\gamma)}(x) = \sum_q Q\,V_\mu^{(q)}(x) \tag{27}$$

$$J_\mu^{(Z)}(x) = \sum_q \left(I_W^3 - 2Q\,\sin^2\theta_W\right)\,V_\mu^{(q)}(x) - I_W^3\,A_\mu^{(q)}(x), \tag{28}$$

$$J_\mu^{(W^\pm)}(x) = \sum_q I_W^\pm\left(V_\mu^{(q)}(x) - A_\mu^{(q)}(x)\right), \tag{29}$$

where the vector and axial vector quark currents are

$$V_\mu^{(q)}(x) = \overline{\psi}(x)\,\gamma_\mu\psi(x), \tag{30}$$

$$A_\mu^{(q)}(x) = \overline{\psi}(x)\,\gamma_\mu\gamma_5\psi(x), \tag{31}$$

One can of course also consider a parametrization of these currents in terms of form factors for a particular flavor, again using hermiticity, parity and the conservation of the vector current. For a spin 1/2 particle (e.g. the nucleon) one obtains

$$\langle P', S'|V_\mu(0)|P, S\rangle = \bar{U}_{S'}(P')\,\underbrace{\left[\gamma_\mu\,F_1^q(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}\,F_2^q(Q^2)\right]}_{\Gamma_\mu^V(P,P')}\,U_S(P), \tag{32}$$

$$\langle P', S'|A_\mu(0)|P, S\rangle = \bar{u}_{S'}(P')\,\underbrace{\left[\gamma_\mu\gamma_5\,G_A^q(Q^2) + \gamma_5\,q_\mu\,G_P^q(Q^2)\right]}_{\Gamma_\mu^A(P,P')}\,U_S(P). \tag{33}$$

Exclusive processes in principle then offer possibilities to measure vector or axial vector form factors for various flavor currents. One can for instance use the (γZ) interference term in electron-nucleon scattering or $\nu_e p \to e^+ n$ processes to separate the currents for different flavors.

Note that the normalizations of the densities for a given flavor imply for the form factor at $Q^2 = 0$,

$$F_1^q(0) = G_E^q(0) = n_q,$$

and one has e.g. $F_1(0) = \sum_q e_q\,n_q = Q$. Note that n_q is the number of quarks minus antiquarks. Other form factors at zero momentum transfer just define some numbers, e.g.

$$F_2^q(0) = \kappa_q, \qquad G_M^q(0) = \mu_q, \qquad G_A^q(0) = g_A^q.$$

One has e.g. $G_M(0) = \sum_q e_q\,\mu_q$. For the axial currents one finds from $\beta-decay$ a proton-neutron transition element of the axial current, that using isospin symmetry can be simply converted into

$$G_A^{p\to n}(0) = g_A^u - g_A^d = 1.26. \tag{34}$$

5. Quark correlation functions in leptoproduction

Within the framework of QCD and knowing that the photon or Z^0 current couples to the quarks, it is possible to write down a diagrammatic expansion for leptoproduction, with in the deep inelastic limit $(Q^2 \to \infty)$ as

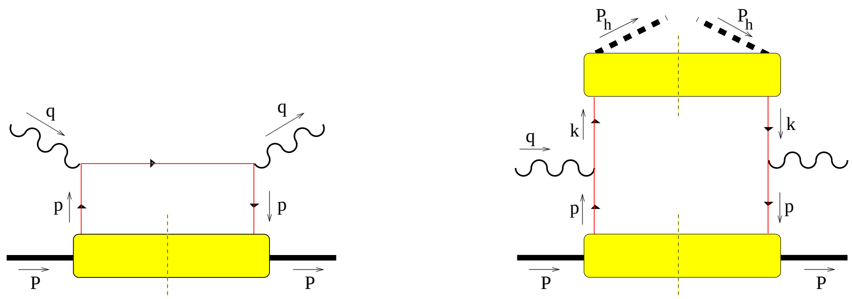


Figure 3. The simplest (parton-level) diagrams representing the squared amplitude in lepton hadron inclusive scattering (left) and semi-inclusive scattering (right). In both cases also the diagram with opposite fermion flow has to be added.

relevant diagrams only the ones given in Fig. 3 for inclusive and 1-particle inclusive scattering respectively. The expression for $\mathcal{W}_{\mu\nu}$ can be rewritten as a nonlocal product of currents and it is a straightforward exercise to show by inserting the currents $j_\mu(x) =: \overline{\psi}(x)\gamma_\mu\psi(x) :$ that for 1-particle inclusive scattering one obtains in tree approximation

$$
\begin{aligned}
&2M\mathcal{W}_{\mu\nu}(q;PS;P_hS_h) \\
&\quad = \frac{1}{(2\pi)^4}\int d^4x\; e^{iq\cdot x}\,\langle PS|:\overline{\psi}_j(x)(\gamma_\mu)_{jk}\psi_k(x):\sum_X |X;P_hS_h\rangle \\
&\qquad\qquad \times\langle X;P_hS_h|:\overline{\psi}_l(0)(\gamma_\nu)_{li}\psi_i(0):|PS\rangle \\
&\quad = \frac{1}{(2\pi)^4}\int d^4x\; e^{iq\cdot x}\,\langle PS|\overline{\psi}_j(x)\psi_i(0)|PS\rangle(\gamma_\mu)_{jk} \\
&\qquad\qquad \langle 0|\psi_k(x)\sum_X |X;P_hS_h\rangle\langle X;P_hS_h|\overline{\psi}_l(0)|0\rangle(\gamma_\nu)_{li} \\
&\qquad + \frac{1}{(2\pi)^4}\int d^4x\; e^{iq\cdot x}\,\langle PS|\psi_k(x)\overline{\psi}_l(0)|PS\rangle(\gamma_\nu)_{li} \\
&\qquad\qquad \langle 0|\overline{\psi}_j(x)\sum_X |X;P_hS_h\rangle\langle X;P_hS_h|\psi_i(0)|0\rangle(\gamma_\mu)_{jk}, \\
&\quad = \int d^4p\,d^4k\,\delta^4(p+q-k)\;\mathrm{Tr}\left(\Phi(p)\gamma_\mu\Delta(k)\gamma_\nu\right) + \left\{\begin{array}{c} q\leftrightarrow -q \\ \mu\leftrightarrow\nu \end{array}\right\},
\end{aligned}
\tag{35}
$$

where

$$
\begin{aligned}
\Phi_{ij}(p) &= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\,\langle PS|\overline{\psi}_j(0)\psi_i(\xi)|PS\rangle, \\
\Delta_{kl}(k) &= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ik\cdot\xi}\,\langle 0|\psi_k(\xi)\sum_X |X;P_hS_h\rangle\langle X;P_hS_h|\overline{\psi}_l(0)|0\rangle.
\end{aligned}
$$

Note that in Φ (quark production) a summation over colors is assumed, while in Δ (quark decay) an averaging over colors is assumed. The quantities Φ and Δ correspond to the blobs in Fig. 3 and parametrize the soft physics. Soft refers to all invariants of momenta being small as compared to the hard scale, i.e. for $\Phi(p)$ one has $p^2 \sim p \cdot P \sim P^2 = M^2 \ll Q^2$.

In general many more diagrams have to be considered in evaluating the hadron tensors, but in the deep inelastic limit they can be neglected or considered as corrections to the soft blobs. We return to this later.

As mentioned above, the relevant structural information for the hadrons is contained in soft parts (the blobs in Fig. 3) which represent specific matrix elements of quark fields. The form of Φ is constrained by hermiticity, parity and time-reversal invariance. The quantity depends besides the quark momentum p on the target momentum P and the spin vector S and one must have

$$[\text{Hermiticity}] \quad \Rightarrow \quad \Phi^\dagger(p,P,S) = \gamma_0\,\Phi(p,P,S)\,\gamma_0, \tag{36}$$

$$[\text{Parity}] \quad \Rightarrow \quad \Phi(p,P,S) = \gamma_0\,\Phi(\bar{p},\bar{P},-\bar{S})\,\gamma_0, \tag{37}$$

$$[\text{Time reversal}] \;\Rightarrow\; \Phi^*(p,P,S) = (-i\gamma_5 C)\,\Phi(\bar{p},\bar{P},\bar{S})\,(-i\gamma_5 C), \tag{38}$$

where $C = i\gamma^2\gamma_0$, $-i\gamma_5 C = i\gamma^1\gamma^3$ and $\bar{p} = (p^0, -\boldsymbol{p})$. The most general way to parametrize Φ using only the constraints from hermiticity and parity invariance, is [2, 3]

$$\begin{aligned}\Phi(p,P,S) \;=\;& M\,A_1 + A_2\,\not{P} + A_3\not{p} + i\,A_4\,\frac{[\not{P},\not{p}]}{2M} \\ &+ i\,A_5\,(p\cdot S)\gamma_5 + M\,A_6\,\not{S}\gamma_5 + A_7\,\frac{(p\cdot S)}{M}\,\not{P}\gamma_5 \\ &+ A_8\,\frac{(p\cdot S)}{M}\,\not{p}\gamma_5 + A_9\,\frac{[\not{P},\not{S}]}{2}\,\gamma_5 + A_{10}\,\frac{[\not{p},\not{S}]}{2}\,\gamma_5 \\ &+ A_{11}\,\frac{(p\cdot S)}{M}\,\frac{[\not{P},\not{p}]}{2M}\,\gamma_5 + A_{12}\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu p^\rho S^\sigma}{M},\end{aligned} \tag{39}$$

where the first four terms do not involve the hadron polarization vector. Hermiticity requires all the amplitudes $A_i = A_i(p\cdot P, p^2)$ to be real. The amplitudes A_4, A_5 and A_{12} vanish when also time reversal invariance applies.

6. Inclusive scattering

6.1. THE RELEVANT SOFT PARTS

In order to find out which information in the soft parts is important in a hard process one needs to realize that the hard scale Q leads in a natural

way to the use of lightlike vectors n_+ and n_- satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$. For inclusive scattering one parametrizes the momenta

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P \cdot q = \frac{Q^2}{x_B} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} q = \frac{Q}{\sqrt{2}}\,n_- \;-\; \frac{Q}{\sqrt{2}}\,n_+ \\ \\ P = \frac{x_B M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \end{array}\right.$$

The above are the external momenta. Next turn to the internal momenta, looking at the left diagram in Fig. 3. In the soft part actually *all* momenta, that is p *and* P have a minus component that can be neglected compared to that in the hard part, since otherwise $p \cdot P$ would be hard. Thus because p must have only a hard plus component, q has two hard components and k being the current jet also must be soft, i.e. only can have one large lightcone component, one must have

$$\begin{aligned} p &= \quad \ldots \quad + \frac{Q}{\sqrt{2}}\,n_+, \\ q &= \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+, \\ p + q = k &= \frac{Q}{\sqrt{2}}\,n_- + \quad \ldots\ . \end{aligned}$$

where the ... parts indicate (negligible) $1/Q$ terms.

Also the transverse component is not relevant for the hard part. One thus sees that for inclusive scattering the only relevant dependence of the soft part is the p^+ dependence. Moreover, the above requirements on the internal momenta already indicate that the lightcone fraction $x = p^+/P^+$ must be equal to x_B. This will come out when we do the actual calculation in one of the next sections.

The minus component $p^- \equiv p \cdot n_+$ and transverse components thus can be integrated over restricting the nonlocality in $\Phi(p)$. The relevant soft part then is some Dirac trace of the quantity [4, 5]

$$\begin{aligned} \Phi_{ij}(x) &= \int dp^-\, d^2p_T\ \Phi_{ij}(p,P,S) \\ &= \int \frac{d\xi^-}{2\pi}\ e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}\,, \end{aligned} \tag{40}$$

depending on the lightcone fraction $x = p^+/P^+$. To be precise one puts in the full form for the quark momentum,

$$p = x\,P^+ n_+ + \frac{p^2 + \boldsymbol{p}_T^2}{2x\,P^+}\,n_- + p_T, \tag{41}$$

and performs the integration over $\Phi(p)$ using

$$\int dp^-\, d^2p_T \ldots = \frac{\pi}{P^+}\int d(p\cdot P)\, dp^2 \ldots\ . \tag{42}$$

When one wants to calculate the leading order in $1/Q$ for a hard process, one only needs to look at leading parts in M/P^+ because $P^+ \propto Q$ (see opening paragraph of this section) [6]. In this case that turns out to be the part proportional to $(M/P^+)^0$,

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\ \not n_+ + \lambda\, g_1(x)\,\gamma_5\ \not n_+ + h_1(x)\,\frac{\gamma_5\,[\,\not S_\perp,\ \not n_+]}{2}\right\} + \mathcal{O}\left(\frac{M}{P^+}\right) \tag{43}$$

The precise expression of the functions $f_1(x)$, etc. as integrals over the amplitudes can be easily written down.

6.2. CALCULATING THE INCLUSIVE CROSS SECTION

Using field theoretical methods the left diagram in Fig. 3 can now be calculated. Omitting the sum over flavors ($\sum_a$), the quark charges e_a^2 and the $(q \leftrightarrow -q, \mu \leftrightarrow \nu)$ 'antiquark' diagram, the symmetric part of the hadron tensor the result is

$$2M\, W^{\mu\nu}(P,q) = \int dp^-\, dp^+\, d^2p_\perp\ \mathrm{Tr}\left(\Phi(p)\,\gamma^\mu \Delta(p+q)\gamma^\nu\right), \tag{44}$$

where

$$\Delta(k) = (\not k + m)\,\delta(k^2-m^2) \approx \frac{\not n_-}{2}\,\delta(k^+), \tag{45}$$

and in the approximation anything proportional to $1/Q^2$ has been neglected. One obtains

$$\begin{aligned} 2M\, W_S^{\mu\nu}(P,q) &= \int dp^-\, dp^+\, d^2p_\perp\, \frac{1}{2}\,\mathrm{Tr}\left(\Phi(p)\,\gamma^\mu\,\gamma^+\,\gamma^\nu\right)\,\delta(p^+ + q^+) \\ &= -g_\perp^{\mu\nu}\ \ \mathrm{Tr}\left(\gamma^+\,\Phi(x)\right)\big|_{x=x_B} \\ &= -g_\perp^{\mu\nu}\, f_1(x_B). \end{aligned} \tag{46}$$

Antiquarks arise from the diagram with opposite fermion flow, proportional to $\mathrm{Tr}\left(\overline{\Phi}(p)\,\gamma^\nu\overline{\Delta}(k)\gamma^\mu\right)$ with

$$\overline{\Phi}_{ij}(p) = \frac{1}{(2\pi)^4}\int d^4\xi\ e^{-ip\cdot\xi}\,\langle PS|\psi_i(\xi)\overline{\psi}_j(0)|PS\rangle. \tag{47}$$

The *proper* definition of antiquark distributions starts from $\Phi^c(x)$ containing antiquark distributions $\bar f_1(x)$, etc. The quantity $\Phi^c(p)$ is obtained from

$\Phi(p)$ after the replacement of ψ by $\psi^C = C\overline{\psi}^T$. One then finds $\overline{\Phi}(p) = -C(\Phi^c)^T C^\dagger$, i.e. one has to be aware of sign differences. Symmetry relations between quark and antiquark relations can be obtained using the anticommutation relations for fermions, giving $\overline{\Phi}_{ij}(p) = -\Phi_{ij}(-p)$. One finds that $\bar{f}_1(x) = -f_1(-x)$, $\bar{g}_1(x) = g_1(-x)$, and $\bar{h}_1(x) = -h_1(-x)$. Finally, after including the flavor summation and the quark charges squared one can compare the result with Eq. 4 to obtain for the structure function

$$2F_1(x_{_B}) = \sum_a e_a^2\,\left(f_1^a(x_{_B}) + f_1^{\bar{a}}(x_{_B})\right), \tag{48}$$

while $F_L(x_{_B}) = 0$ (Callan-Gross relation).

The antisymmetric part of $W^{\mu\nu}$ in the above calculation is left as an exercise. The answer is

$$2M\,W_A^{\mu\nu}(P,q) \quad = \quad i\,\epsilon_\perp^{\mu\nu}\,g_1(x_{_B}), \tag{49}$$

which after inclusion of antiquarks, flavor summation gives after comparison with Eq. 12

$$2g_1(x_{_B}) = \sum_a e_a^2\,\left(g_1^a(x_{_B}) + g_1^{\bar{a}}(x_{_B})\right). \tag{50}$$

6.3. INTERPRETATION OF THE FUNCTIONS

The functions f_1, g_1 and h_1 can be obtained from the correlator $\Phi(x)$ after tracing with the appropriate Dirac matrix,

$$f_1(x) \quad = \quad \int \frac{d\xi^-}{4\pi}\;e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\gamma^+\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}, \tag{51}$$

$$\lambda\,g_1(x) \quad = \quad \int \frac{d\xi^-}{4\pi}\;e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\gamma^+\gamma_5\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}, \tag{52}$$

$$S_T^i\,h_1(x) \quad = \quad \int \frac{d\xi^-}{4\pi}\;e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,i\sigma^{i+}\gamma_5\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}, \tag{53}$$

By introducing *good* and *bad* fields $\psi_\pm \equiv \frac{1}{2}\gamma^\mp\gamma^\pm\psi$, one sees that f_1 can be rewritten as

$$\begin{aligned} f_1(x) \quad &= \quad \int \frac{d\xi^-}{2\pi\sqrt{2}}\;e^{ip\cdot\xi}\,\langle P,S|\psi_+^\dagger(0)\psi_+(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0} \\ &= \quad \frac{1}{\sqrt{2}}\sum_n |\langle P_n|\psi_+|P\rangle|^2\;\delta\left(P_n^+ - (1-x)P^+\right), \end{aligned} \tag{54}$$

i.e. it is a quark lightcone momentum distribution. For the functions g_1 and h_1 one needs in addition the projectors on quark chirality states, $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$, and on quark transverse spin states [7, 6], $P_{\uparrow/\downarrow} = \frac{1}{2}(1 \pm \gamma^i \gamma_5)$ to see that

$$f_1(x) = f_{1R}(x) + f_{1L}(x) = f_{1\uparrow}(x) + f_{1\downarrow}(x), \tag{55}$$

$$g_1(x) = f_{1R}(x) - f_{1L}(x), \tag{56}$$

$$h_1(x) = f_{1\uparrow}(x) - f_{1\downarrow}(x). \tag{57}$$

One sees some trivial bounds such as $f_1(x) \geq 0$ and $|g_1(x)| \leq f_1(x)$. Since $P_n^+ \leq 0$ and sees $x \leq 1$. From the antiquark distribution $\bar{f}_1(x)$ and its relation to $f_1(x)$ one obtains $x \geq -1$, thus the support of the functions is $-1 \leq x \leq 1$.

6.4. BOUNDS ON THE DISTRIBUTION FUNCTIONS

The trivial bounds on the distribution functions ($|h_1(x)| \leq f_1(x)$ and $|g_1(x)| \leq f_1(x)$) can be sharpened. For instance one can look explicitly at the structure in Dirac space of the correlation function Φ_{ij}. Actually, we will look at the correlation functions $(\Phi\,\gamma_0)_{ij}$, which involves at leading order matrix elements $\psi^\dagger_{+j}(0)\psi_{+i}(\xi)$. One has in Weyl representation ($\gamma^0 = \rho^1$, $\gamma^i = -i\rho^2\sigma^i$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \rho^3$) the matrices

$$P_+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$P_+\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad P_+\gamma^1\gamma_5 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

The good projector only leaves two (independent) Dirac spinors, one righthanded (R), one lefthanded (L). On this basis of good R and L spinors the for hard scattering processes relevant matrix $(\Phi \not{n}_-)$ is given by

$$(\Phi\,\not{n}_-)_{ij}(x) = \begin{pmatrix} f_1 + \lambda\, g_1 & (S_T^1 - i\, S_T^2)\, h_1 \\ & \\ (S_T^1 + i\, S_T^2)\, h_1 & f_1 - \lambda\, g_1 \end{pmatrix} \tag{58}$$

One can also turn the S-dependent correlation function Φ defined in analogy with $W(q,P,S)$ in Eq. 10 into a matrix in the nucleon spin space. If

$$\Phi(x;P,S) = \Phi_O + \lambda\,\Phi_L + S_T^1\,\Phi_T^1 + S_T^2\,\Phi_T^2, \tag{59}$$

then one has on the basis of spin 1/2 target states with $\lambda = +1$ and $\lambda = -1$ respectively

$$\Phi_{ss'}(x) = \begin{pmatrix} \Phi_O + \Phi_L & \Phi_T^1 - i\,\Phi_T^2 \\ \Phi_T^1 + i\,\Phi_T^2 & \Phi_O - \Phi_L \end{pmatrix} \tag{60}$$

The matrix relevant for bounds is the matrix $M = (\Phi \not{n}_-)^T$ (for this matrix one has $v^\dagger M v \geq 0$ for any direction v). On the basis $+R$, $-R$, $+L$ and $-L$ it becomes

$$(\Phi(x)\not{n}_-)^T = \begin{pmatrix} f_1 + g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1 + g_1 \end{pmatrix}. \tag{61}$$

Of this matrix any diagonal matrix element must always be positive, hence the eigenvalues must be positive, which gives a bound on the distribution functions stronger than the trivial bounds, namely

$$|h_1(x)| \leq \frac{1}{2}\,(f_1(x) + g_1(x)) \tag{62}$$

known as the Soffer bound [8].

6.5. SUM RULES

For the functions appearing in the soft parts, and thus also for the structure functions, one can derive sum rules. Starting with the traces defining the quark distributions,

$$\begin{aligned} f_1(x) &= \int \frac{d\xi^-}{4\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\gamma^+\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}, \\ g_1(x) &= \int \frac{d\xi^-}{4\pi}\, e^{ip\cdot\xi}\, \langle P,S|\overline{\psi}(0)\gamma^+\gamma_5\psi(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}, \end{aligned}$$

and integrating over $x = p^+/P^+$ one obtains (using symmetry relation as indicated above to eliminate antiquarks $\bar{f}_1$),

$$\int_0^1 dx\ (f_1(x) - \bar{f}_1(x)) = \int_{-1}^1 dx\ f_1(x) = \frac{\langle P,S|\overline{\psi}(0)\gamma^+\psi(0)|P,S\rangle}{2P^+}, \tag{63}$$

which as we have seen in the section on elastic scattering is nothing else than a form factor at zero momentum transfer, i.e. the number of quarks of that particular flavor. Similarly one finds the sum rule

$$\int_0^1 dx\ (g_1(x) + \bar{g}_1(x)) = \int_{-1}^1 dx\ g_1(x) = \frac{\langle P, S|\overline{\psi}(0)\gamma^+\gamma_5\psi(0)|P, S\rangle}{2P^+}, \quad (64)$$

which precisely is the axial charge g_A for a particular quark flavor. These sum rules for the quark distributions underly the sum rules for the structure functions, e.g. the Bjorken sum rule following from Eq. 64 and Eq. 34.

$$\int_0^1 dx_{_B}\ \left(g_1^p(x_{_B}, Q^2) - g_1^n(x_{_B}, Q^2)\right) = \frac{1}{6}\left(g_A^u - g_A^d\right) = \frac{1}{6}\, G_A^{p\rightarrow n}(0). \quad (65)$$

7. 1-particle inclusive scattering

7.1. THE RELEVANT DISTRIBUTION FUNCTIONS

For 1-particle inclusive scattering one parametrizes the momenta

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ P_h^2 = M_h^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \\ 2\,P_h\cdot q = -z_h\,Q^2 \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P_h = \frac{z_h\,Q}{\sqrt{2}}\,n_- + \frac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\ \\ q = \frac{Q}{\sqrt{2}}\,n_- \ - \ \frac{Q}{\sqrt{2}}\,n_+ \ + \ q_T \\ \\ P = \frac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \end{array}\right.$$

Note that this works for socalled current fragmentation, in which case the produced hadron is *hard* with respect to the target momentum, i.e. $P\cdot P_h \sim Q^2$. The minus component p^- is irrelevant in the lower soft part, while the plus component k^+ is irrelevant in the upper soft part. Note that after the choice of P and P_h one can no longer omit a transverse component in the other vector, in this case the momentum transfer q. This is precisely the vector q_T introduced earlier in the discussion of the structure functions for 1-particle inclusive leptoproduction. One immediately sees that one can no longer simply integrate over the transverse component of the quark momentum, defined in Eq. 41.

At this point it turns out that the most convenient way to describe the spin vector of the target is via an expansion of the form

$$S^\mu = -\lambda\,\frac{M x_{_B}}{Q\sqrt{2}}\,n_- + \lambda\,\frac{Q}{M x_{_B}\sqrt{2}}\,n_+ + S_{_T}. \quad (66)$$

One has up to $\mathcal{O}(1/Q^2)$ corrections $\lambda \approx M\,(S\cdot q)/(P\cdot q)$ and $S_T \approx S_\perp$. For a pure state one has $\lambda^2 + \boldsymbol{S}_T^2 = 1$, in general this quantity being less or equal than one.

The soft part to look at is

$$\Phi(x,\boldsymbol{p}_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}. \tag{67}$$

For the leading order results, it is parametrized as

$$\Phi(x,\boldsymbol{p}_T) = \Phi_O(x,\boldsymbol{p}_T) + \Phi_L(x,\boldsymbol{p}_T) + \Phi_T(x,\boldsymbol{p}_T), \tag{68}$$

with the parts involving unpolarized targets (O), longitudinally polarized targets (L) and transversely polarized targets (T) up to parts proportional to M/P^+ given by

$$\Phi_O(x,\boldsymbol{p}_T) = \frac{1}{2}\left\{ f_1(x,\boldsymbol{p}_T)\; \not{n}_+ + h_1^\perp(x,\boldsymbol{p}_T)\,\frac{i\,[\not{p}_T,\,\not{n}_+]}{2M}\right\} \tag{69}$$

$$\Phi_L(x,\boldsymbol{p}_T) = \frac{1}{2}\left\{ \lambda\, g_{1L}(x,\boldsymbol{p}_T)\,\gamma_5\; \not{n}_+ + \lambda\, h_{1L}^\perp(x,\boldsymbol{p}_T)\frac{\gamma_5\,[\not{p}_T,\,\not{n}_+]}{2M}\right\} \tag{70}$$

$$\begin{aligned}\Phi_T(x,\boldsymbol{p}_T) = \frac{1}{2}\Bigg\{ & f_{1T}^\perp(x,\boldsymbol{p}_T)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} \\ & + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\, g_{1T}(x,\boldsymbol{p}_T)\,\gamma_5\; \not{n}_+ + h_{1T}(x,\boldsymbol{p}_T)\,\frac{\gamma_5\,[\not{S}_T,\,\not{n}_+]}{2} \\ & + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\, h_{1T}^\perp(x,\boldsymbol{p}_T)\,\frac{\gamma_5\,[\not{p}_T,\,\not{n}_+]}{2M}\Bigg\}. \end{aligned} \tag{71}$$

All functions appearing here have a natural interpretation as densities. This is seen as discussed before for the $\boldsymbol{p}_T$-integrated functions. Now it includes densities such as the density of longitudinally polarized quarks in a transversely polarized nucleon (g_{1T}) and the density of transversely polarized quarks in a longitudinally polarized nucleon ($h_{1L}^\perp$). The interpretation of all functions is illustrated in Fig. 4.

Several functions vanish from the soft part upon integration over p_T. Actually we will find that particularly interesting functions survive when one integrates over $\boldsymbol{p}_T$ weighting with p_T^α, e.g.

$$\begin{aligned}\Phi_\partial^\alpha(x) &\equiv \int d^2p_T\,\frac{p_T^\alpha}{M}\,\Phi(x,\boldsymbol{p}_T) \\ &= \frac{1}{2}\Bigg\{ -g_{1T}^{(1)}(x)\,S_T^\alpha\,\not{n}_+\gamma_5 - \lambda\, h_{1L}^{\perp(1)}(x)\,\frac{[\gamma^\alpha,\not{n}_+]\gamma_5}{2} \\ &\qquad - f_{1T}^{\perp(1)}\,\epsilon^\alpha{}_{\mu\nu\rho}\gamma^\mu n_-^\nu S_T^\rho - h_1^{\perp(1)}\,\frac{i[\gamma^\alpha,\,\not{n}_+]}{2}\Bigg\}, \end{aligned} \tag{72}$$

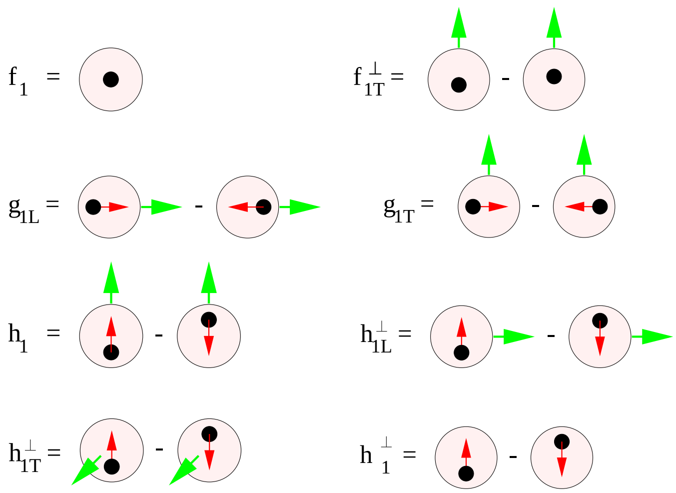


Figure 4. Interpretation of the functions in the leading Dirac traces of Φ.

where we define $\boldsymbol{p}_T^2/2M^2$-moments as

$$g_{1T}^{(1)}(x) = \int d^2p_T\ \frac{\boldsymbol{p}_T^2}{2M^2}\, g_{1T}(x, \boldsymbol{p}_T), \tag{73}$$

and similarly the other functions. The functions $h_1^{\perp}$ and $f_{1T}^{\perp}$ are T-odd, vanishing if T-reversal invariance can be applied to the matrix element. For p_T-dependent correlation functions, matrix elements involving gluon fields at infinity (gluonic poles [9]) can for instance prevent application of T-reversal invariance. The functions describe the possible appearance of unpolarized quarks in a transversely polarized nucleon ($f_{1T}^{\perp}$) or transversely polarized quarks in an unpolarized hadron ($h_1^{\perp}$) and lead to single-spin asymmetries in various processes [10, 11]. The interpretation of these functions is also illustrated in Fig. 4. Of course just integrating $\Phi(x, p_T)$ over p_T gives the result used in inclusive scattering with $f_1(x) = \int d^2p_T\ f_1(x, p_T)$, $g_1(x) = g_{1L}(x)$ and $h_1(x) = h_{1T}(x) + h_{1T}^{\perp(1)}(x)$. We note that the function $h_{1T}^{\perp(2)}$ appears after weighting with $p_T^{\alpha} p_T^{\beta}$.

7.2. THE RELEVANT FRAGMENTATION FUNCTIONS

Just as for the distribution functions one can perform an analysis of the soft part describing the quark fragmentation. One needs [12]

$$\Delta_{ij}(z, \boldsymbol{k}_T) = \sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, Tr\langle 0|\psi_i(\xi)|P_h, X\rangle\langle P_h, X|\overline{\psi}_j(0)|0\rangle\Big|_{\xi^-=0}. \tag{74}$$

For the production of unpolarized hadrons h in hard processes one needs to leading order in $1/Q$ the correlation function,

$$\Delta_O(z, \boldsymbol{k}_T) = z\, D_1(z, \boldsymbol{k}'_T)\; \not{n}_- + z\, H_1^\perp(z, \boldsymbol{k}'_T)\, \frac{i\,[\not{k}_T, \not{n}_-]}{2M_h} + \mathcal{O}\left(\frac{M_h}{P_h^-}\right). \quad (75)$$

when we limit ourselves to an unpolarized or spin 0 final state hadron. The arguments of the fragmentation functions D_1 and $H_1^\perp$ are $z = P_h^-/k^-$ and $\boldsymbol{k}'_T = -z\boldsymbol{k}_T$. The first is the (lightcone) momentum fraction of the produced hadron, the second is the transverse momentum of the produced hadron with respect to the quark. The fragmentation function D_1 is the equivalent of the distribution function f_1. It can be interpreted as the probability of finding a hadron h in a quark. The function $H_1^\perp$, interpretable as the difference in production probabilities of unpolarized hadrons from a transversely polarized quark depending on transverse momentum, is allowed because of the non-applicability of time reversal invariance [13]. This is natural for the fragmentation functions [14, 15] because of the appearance of out-states $|P_h, X\rangle$ in the definition of Δ, in contrast to the plane wave states appearing in Φ. After $\boldsymbol{k}_T$-averaging one is left with the functions $D_1(z)$ and the $\boldsymbol{k}_T^2/2M^2$-weighted result $H_1^{\perp(1)}(z)$.

7.3. THE SEMI-INCLUSIVE CROSS SECTION

After the analysis of the soft parts, the next step is to find out how one obtains the information on the various correlation functions from experiments, in this particular case in lepton-hadron scattering via one-photon exchange as discussed before. To get the leading order result for semi-inclusive scattering it is sufficient to compute the diagram in Fig. 3 (right) by using QCD and QED Feynman rules in the hard part and the matrix elements Φ and Δ for the soft parts, parametrized in terms of distribution and fragmentation functions. The most well-known results for leptoproduction are:

$$\frac{d\sigma_{OO}}{dx_B\, dy\, dz_h} = \frac{2\pi\alpha^2\, s}{Q^4} \sum_{a,\bar{a}} e_a^2\, \Big(1 + (1-y)^2\Big)\; x_B f_1^a(x_B)\, D_1^a(z_h) \quad (76)$$

$$\frac{d\sigma_{LL}}{dx_B\, dy\, dz_h} = \frac{2\pi\alpha^2\, s}{Q^4}\, \lambda_e\, \lambda \sum_{a,\bar{a}} e_a^2\; y(2-y)\; x_B g_1^a(x_B)\, D_1^a(z_h) \quad (77)$$

The indices attached to the cross section refer to polarization of lepton (O is unpolarized, L is longitudinally polarized) and hadron (O is unpolarized, L is longitudinally polarized, T is transversely polarized). Note that the result is a weighted sum over quarks and antiquarks involving the charge e_a squared. Comparing with well-known formal expansions of the cross

section in terms of structure functions one can simply identify these. For instance the above result for unpolarized scattering (OO) shows that after averaging over azimuthal angles, only one structure function survives if we work at order α_s^0 and at leading order in $1/Q$.

As we have seen, in 1-particle inclusive unpolarized leptoproduction in principle four structure functions appear, two of them containing azimuthal dependence of the form $\cos(\phi_h^\ell)$ and $\cos(2\phi_h^\ell)$. The first one only appears at order $1/Q$ [16], the second one even at leading order but only in the case of the existence of nonvanishing T-odd distribution functions. To be specific if we define weighted cross section such as

$$\int d^2\boldsymbol{q}_T \, \frac{Q_T^2}{M M_h} \, \cos(2\phi_h^\ell) \, \frac{d\sigma_{OO}}{dx_{_B}\, dy\, dz_h\, d^2\boldsymbol{q}_T} \equiv \left\langle \frac{Q_T^2}{M M_h} \, \cos(2\phi_h^\ell) \right\rangle_{OO} \tag{78}$$

we obtain the following asymmetry,.

$$\left\langle \frac{Q_T^2}{M M_h} \, \cos(2\phi_h^\ell) \right\rangle_{OO} = \frac{16\pi\alpha^2\, s}{Q^4} \, (1-y) \sum_{a,\bar{a}} e_a^2 \, x_{_B} \, h_1^{\perp(1)a}(x_{_B}) H_1^{\perp(1)a}. \tag{79}$$

In lepton-hadron scattering this asymmetry requires T-odd distribution functions and therefore most likely is absent or very small. In e^+e^- annihilation, however, a $\cos 2\phi$ asymmetry between produced particles (e.g. pions) in opposite jets involves two very likely nonvanishing fragmentation functions $H_1^\perp$ and $\overline{H}_1^\perp$. Indications for the presence of these fragmentation functions have been found in LEP data[17].

For polarized targets, several azimuthal asymmetries arise already at leading order. For example the following possibilities were investigated in Refs [18, 13, 19, 20].

$$\left\langle \frac{Q_{_T}}{M} \, \cos(\phi_h^\ell - \phi_S^\ell) \right\rangle_{LT} = \\ \frac{2\pi\alpha^2\, s}{Q^4} \, \lambda_e \, |\boldsymbol{S}_{_T}| \, y(2-y) \sum_{a,\bar{a}} e_a^2 \, x_{_B} \, g_{1T}^{(1)a}(x_{_B}) D_1^a(z_h), \tag{80}$$

$$\left\langle \frac{Q_{_T}^2}{M M_h} \, \sin(2\phi_h^\ell) \right\rangle_{OL} = \\ -\frac{4\pi\alpha^2\, s}{Q^4} \, \lambda \, (1-y) \sum_{a,\bar{a}} e_a^2 \, x_{_B} \, h_{1L}^{\perp(1)a}(x_{_B}) H_1^{\perp(1)a}(z_h), \tag{81}$$

$$\left\langle \frac{Q_{_T}}{M_h} \, \sin(\phi_h^\ell + \phi_S^\ell) \right\rangle_{OT} = \\ \frac{4\pi\alpha^2\, s}{Q^4} \, |\boldsymbol{S}_{_T}| \, (1-y) \sum_{a,\bar{a}} e_a^2 \, x_{_B} \, h_1^a(x_{_B}) H_1^{\perp(1)a}(z_h). \tag{82}$$

The latter two are single spin asymmetries involving the fragmentation function $H_1^{\perp(1)}$. The last one was the asymmetry proposed by Collins [13] as a way to access the transverse spin distribution function h_1 in pion production. Note, however, that in using the azimuthal dependence one needs to be very careful. For instance, besides the $<\sin(\phi_h^\ell + \phi_S^\ell)>_{OT}$, one also finds at leading order a $<\sin(3\phi_h^\ell - \phi_S^\ell)>_{OT}$ asymmetry which is proportional to $h_{1T}^{\perp(2)}\, H_1^{\perp(1)}$ [20].

8. Inclusion of subleading contributions

8.1. SUBLEADING INCLUSIVE LEPTOPRODUCTION

If one proceeds up to order $1/Q$ one also needs terms in the parametrization of the soft part proportional to M/P^+. Limiting ourselves to the $\boldsymbol{p}_T$-integrated correlations one needs

$$\begin{aligned}
\Phi(x) &= \frac{1}{2}\left\{ f_1(x)\, \not{n}_+ + \lambda\, g_1(x)\, \gamma_5\, \not{n}_+ + h_1(x)\, \frac{\gamma_5\, [\, \not{S}_T,\, \not{n}_+]}{2} \right\} \\
&+ \frac{M}{2P^+}\left\{ e(x) + g_T(x)\, \gamma_5\, \not{S}_T + \lambda\, h_L(x)\, \frac{\gamma_5\, [\, \not{n}_+,\, \not{n}_-]}{2} \right\} \\
&+ \frac{M}{2P^+}\left\{ -\lambda\, e_L(x)\, i\gamma_5 - f_T(x)\, \epsilon_T^{\rho\sigma}\, \gamma_\rho S_{T\sigma} + h(x)\, \frac{i\, [\, \not{n}_+,\, \not{n}_-]}{2} \right\} \quad (83)
\end{aligned}$$

The last set of three terms proportional to M/P^+ vanish when time-reversal invariance applies.

Actually in the calculation of the cross section one has to be careful. Let us use inclusive scattering off a transversely polarized nucleon (transverse means $|\boldsymbol{S}_\perp| = 1$ in Eq. 11) as an example. The hadronic tensor is zero in leading order in $1/Q$. At order $1/Q$ one obtains from the handbag diagram a contribution

$$2M\, W_{A(a)}^{\mu\nu}(q, P, S_T) = i\, \frac{2M}{Q}\, \hat{t}^{\,[\mu} \epsilon_\perp^{\nu]\rho} S_{\perp\rho} \left(g_{1T}^{(1)}(x_B) - \frac{m}{M}\, h_1(x_B) \right). \quad (84)$$

It shows that one must be very careful with the integration over p_T.

There is a second contribution at order $1/Q$ coming from diagrams as the one shown in Fig. 5. For these gluon diagrams one needs matrix elements containing $\overline{\psi}(0)\, gA_T^\alpha(\eta)\, \psi(\xi)$. At order $1/Q$ one only needs the matrix element of the bilocal combinations $\overline{\psi}(0)\, gA_T^\alpha(\xi)\, \psi(\xi)$ and $\overline{\psi}(0)\, gA_T^\alpha(0)\, \psi(\xi)$ These soft parts have a structure quite similar to Φ_∂^α and are parametrized as

$$\Phi_A^\alpha(x) = \frac{M}{2}\left\{ -x\, \tilde{g}_T(x)\, S_T^\alpha\, \not{n}_+ \gamma_5 - \lambda\, x\, \tilde{h}_L(x)\, \frac{[\gamma^\alpha, \not{n}_+]\gamma_5}{2} \right.$$

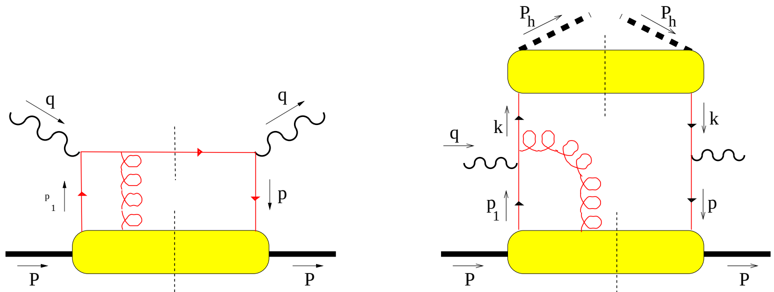


Figure 5. Examples of gluonic diagrams that must be included at subleading order in lepton hadron inclusive scattering (left) and in semi-inclusive scattering (right).

$$- x\,\tilde{f}_T(x)\,\epsilon^{\alpha}{}_{\mu\nu\rho}\gamma^{\mu}n_-^{\nu}S_T^{\rho} - x\,\tilde{h}(x)\,\frac{i[\gamma^{\alpha},\,\not{n}_+]}{2}\Bigg\}. \tag{85}$$

This contributes also to $W_A^{\mu\nu}$,

$$2M\,W^{\mu\nu}_{A(b)}(q,P,S_T) = i\,\frac{2Mx_B}{Q}\,\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,\tilde{g}_T(x_B). \tag{86}$$

Using the QCD equations of motion, however, these functions can be related to the functions appearing in Φ. To be precise one combines $i\partial$ in Φ_∂ (see Eq. 72) and A_μ in Φ_A to Φ_D containing $iD_\mu = i\partial_\mu + g\,A_\mu$ for which one has via the equations of motion

$$\begin{aligned}
\Phi_D^{\alpha}(x) \;=\; \frac{M}{2}\Bigg\{ & -\left(x\,g_T - \frac{m}{M}\,h_1\right)\,S_T^{\alpha}\,\not{n}_+\gamma_5 \\
& - \lambda\left(x\,h_L - \frac{m}{M}\,g_1\right)\,\frac{[\gamma^{\alpha},\not{n}_+]\gamma_5}{2} \\
& - x\,f_T(x)\,\epsilon^{\alpha}{}_{\mu\nu\rho}\gamma^{\mu}n_-^{\nu}S_T^{\rho} - x\,\tilde{h}(x)\,\frac{i[\gamma^{\alpha},\,\not{n}_+]}{2}\Bigg\}.
\end{aligned} \tag{87}$$

Hence one obtains

$$x\,\tilde{g}_T = x\,g_T - g_{1T}^{(1)} - \frac{m}{M}\,h_1, \tag{88}$$

$$x\,\tilde{h}_L = x\,h_L - h_{1L}^{\perp(1)} - \frac{m}{M}\,g_1, \tag{89}$$

$$x\,\tilde{f}_T = x\,f_T + f_{1T}^{(1)}, \tag{90}$$

$$x\,\tilde{h} = x\,h + 2\,h_1^{\perp(1)}. \tag{91}$$

and one obtains the full contribution

$$2M\,W_A^{\mu\nu}(q,P,S_T) = i\,\frac{2Mx_B}{Q}\,\hat{t}^{\,[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,g_T(x_B), \tag{92}$$

leading for the structure function $g_T(x_B,Q^2)$ defined in Eq. 12 to the result

$$g_T(x_B,Q^2) = \frac{1}{2}\sum_a e_a^2\left(g_T^a(x_B) + g_T^{\bar{a}}(x_B)\right). \tag{93}$$

From Lorentz invariance one obtains, furthermore, some interesting relations between the subleading functions and the k_T-dependent leading functions [21, 22, 23]. Just by using the expressions for the functions in terms of the amplitudes A_i in Eq. 39 one finds

$$g_T = g_1 + \frac{d}{dx}\,g_{1T}^{(1)}, \tag{94}$$

$$h_L = h_1 - \frac{d}{dx}\,h_{1L}^{\perp(1)}, \tag{95}$$

$$f_T = -\frac{d}{dx}\,f_{1T}^{\perp(1)}, \tag{96}$$

$$h = -\frac{d}{dx}\,h_1^{\perp(1)}. \tag{97}$$

As an application, one can eliminate $g_{1T}^{(1)}$ using Eq. 94 and obtain (assuming sufficient neat behavior of the functions) for $g_2 = g_T - g_1$

$$\begin{aligned} g_2(x) \;\;=\;\; & -\left[g_1(x) - \int_x^1 dy\,\frac{g_1(y)}{y}\right] + \frac{m}{M}\left[\frac{h_1(x)}{x} - \int_x^1 dy\,\frac{h_1(y)}{y^2}\right] \\ & + \left[\tilde{g}_T(x) - \int_x^1 dy\,\frac{\tilde{g}_T(y)}{y}\right]. \end{aligned} \tag{98}$$

One can use this to obtain for each quark flavor $\int dx\,g_2^a(x) = 0$, the Burkhardt-Cottingham sumrule [24]. Neglecting the interaction-dependent part one obtains the Wandzura-Wilczek approximation [25] for g_2, which in particular when one neglects the quark mass term provides a simple and often used estimate for g_2. It has become the standard with which experimentalists compare the results for g_2.

Actually the SLAC results for g_2 can also be used to estimate the function $g_{1T}^{(1)}$ and the resulting asymmetries, e.g. the one in Eq. 80. For this one needs the exact relation in Eq. 94. Results can be found in Refs [18] and [26].

8.2. SUBLEADING 1-PARTICLE INCLUSIVE LEPTOPRODUCTION

Also for the transverse momentum dependent functions dependent distribution and fragmentation functions one can proceed to subleading order [22]. We will not discuss these functions here.

In semi-inclusive cross sections one also needs fragmentation functions, for which similar relations exist, e.g. the relation in Eq. 97 for distribution functions has an analog for fragmentation functions, relating $H_1^{\perp(1)}$ (appearing in Eqs 81 and 82) and an at subleading order appearing function $H(z)$,

$$\frac{H(z)}{z} = z^2\,\frac{d}{dz}\left(\frac{H_1^{\perp(1)}}{z}\right). \tag{99}$$

An interesting subleading asymmetry involving $H_1^\perp$ is a $\sin(\phi_h^\ell)$ single spin asymmetry appearing as the structure functions $\mathcal{H}'_{LT}$ in Eq. sidiswanti for a polarized lepton but unpolarized target [16],

$$\begin{aligned}\left\langle \frac{Q_T}{M}\,\sin(\phi_h^\ell)\right\rangle_{LO} &= \\ &\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,y\sqrt{1-y}\,\frac{2M}{Q}\,x_{_B}^2\,\tilde{e}^a(x_{_B})\,H_1^{\perp(1)a}(z_h)\end{aligned} \tag{100}$$

where $\tilde{e}^a(x) = e^a(x) - (m_a/M)\,(f_1^a(x)/x)$. This cross section involves, besides the time-reversal odd fragmentation function $H_1^\perp$, the distribution function e. The tilde function that appear in the cross sections is in fact the socalled interaction dependent part of the twist three functions. It would vanish in any naive parton model calculation in which cross sections are obtained by folding electron-parton cross sections with parton densities. Considering the relation for $\tilde{e}$ one can state it as $x\,e(x) = (m/M)\,f_1(x)$ in the absence of quark-quark-gluon correlations. The inclusion of the latter also requires diagrams dressed with gluons as shown in Fig. 5.

9. Color gauge invariance

We have sofar neglected two problems. The first problem is that the correlation function Φ discussed in previous sections involve two quark fields at different space-time points and hence are not color gauge invariant. The second problem comes from the gluonic diagrams similar as the ones we have discussed in the previous section (see Fig. 5) We note that diagrams involving matrix elements with longitudinal (A^+) gluon fields,

$$\overline{\psi}_j(0)\,gA^+(\eta)\,\psi_i(\xi)$$

do not lead to any suppression. The reason is that because of the +-index in the gluon field the matrix element is proportional to P^+, p^+ or $M\,S^+$

rather than the proportionality to $M\,S_T^\alpha$ or p_T^α that we have seen in Eq. 85 for a gluonic matrix element with transverse gluons.

A straightforward calculation, however, shows that the gluonic diagrams with one or more longitudinal gluons involve matrix elements (soft parts) of operators $\overline{\psi}\psi$, $\overline{\psi}\,A^+\,\psi$, $\overline{\psi}\,A^+A^+\,\psi$, etc. that can be resummed into a correlation function

$$\Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)|P,S\rangle\bigg|_{\xi^+=\xi_T=0}, \tag{101}$$

where $\mathcal{U}$ is a gauge link operator

$$\mathcal{U}(0,\xi) = \mathcal{P}\exp\left(-i\int_0^{\xi^-} d\zeta^-\;A^+(\zeta)\right) \tag{102}$$

(path-ordered exponential with path along $-$-direction). Et voila, the unsuppressed gluonic diagrams combine into a color gauge invariant correlation function. We note that at the level of operators, one expands

$$\overline{\psi}(0)\psi(\xi) = \sum_n \frac{\xi^{\mu_1}\ldots\xi^{\mu_n}}{n!}\,\overline{\psi}(0)\partial_{\mu_1}\ldots\partial_{\mu_n}\psi(0), \tag{103}$$

in a set of local operators, but only the expansion of the nonlocal combination with a gauge link

$$\overline{\psi}(0)\psi(\xi) = \sum_n \frac{\xi^{\mu_1}\ldots\xi^{\mu_n}}{n!}\,\overline{\psi}(0)D_{\mu_1}\ldots D_{\mu_n}\psi(0), \tag{104}$$

is an expansion in terms of local gauge invariant operators. The latter operators are precisely the local (quark) operators that appear in the operator product expansion applied to inclusive deep inelastic scattering.

For the p_T-dependent functions, one finds that inclusion of A^+ gluonic diagrams leads to a color gauge invariant matrix element with links running via $\xi^= = \pm\infty$. For instance in lepton-hadron scattering one finds

$$\Phi(x,\boldsymbol{p}_T) = \int \frac{d\xi^-d^2\boldsymbol{\xi}_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\overline{\psi}(0)\,\mathcal{U}(0,\infty)\,\mathcal{U}(\infty,\xi)\,\psi(\xi)|P,S\rangle\bigg|_{\xi^+=0}, \tag{105}$$

where the gauge links are at constant ξ_T. One can multiply this correlator with p_T^α and make this into a derivative ∂_α. Including the links one finds the color gauge invariant result

$$p_T^\alpha\,\Phi_{ij}(x,\boldsymbol{p}_T) = (\Phi_\partial^\alpha)_{ij}(x,\boldsymbol{p}_T)$$

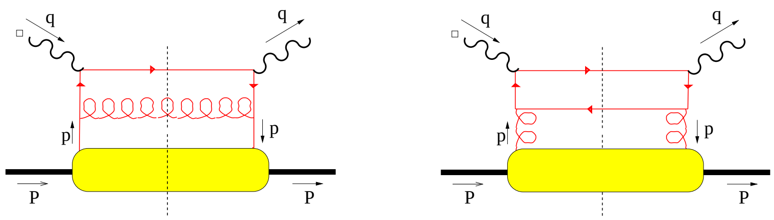


Figure 6. Ladder diagrams used to calculate the asymptotic behavior of the correlation functions.

$$
\begin{aligned}
= \int \frac{d\xi^- \, d^2\boldsymbol{\xi}_T}{(2\pi)^3} \; e^{ip\cdot\xi} \Big\{ & \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\infty)\, iD_T^\alpha \psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0} \\
& - \langle P,S|\overline{\psi}_j(0)\,\mathcal{U}(0,\infty) \int_\infty^{\xi^-} d\eta^- \, \mathcal{U}(\infty,\eta) \\
& \quad \times g\, G^{+\alpha}(\eta)\,\mathcal{U}(\eta,\xi)\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0}\Big\},
\end{aligned} \tag{106}
$$

which gives after integration over p_T the expected result $\Phi_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)$. Note that in $A^+ = 0$ gauge all the gauge links disappear, while one has $G^{+\alpha} = \partial^+ A^\alpha$, but there presence is essential to perform the above differentiations.

10. Evolution

The explicit treatment of transverse momenta provides also a transparent way to include the evolution equations for quark distribution and fragmentation functions. Remember that we have assumed that soft parts vanish sufficiently fast as a function of the invariants $p\cdot P$ and p^2, which at constant x implies a sufficiently fast vanishing as a function of $\boldsymbol{p}_T^2$. This simply turns out not to be true. Assuming that the result for $\boldsymbol{p}_T^2 \geq \mu^2$ is given by the diagram shown in Fig. 6 one finds

$$
\begin{aligned}
f_1(x,\boldsymbol{p}_T^2) &= \theta(\mu^2 - \boldsymbol{p}_T^2)\, f_1(x,\boldsymbol{p}_T^2) \\
&+ \theta(\boldsymbol{p}_T^2 - \mu^2)\, \frac{1}{\pi\,\boldsymbol{p}_T^2}\, \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y}\, P_{qq}\left(\frac{x}{y}\right) f_1(y;\mu^2),
\end{aligned} \tag{107}
$$

where $f_1(x;\mu^2) = \pi \int_0^{\mu^2} d\boldsymbol{p}_T^2 \; f_1(x,\boldsymbol{p}_T^2)$ and the splitting function is given by

$$
P_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z)\right], \tag{108}
$$

with $\int dz\, f(z)/(1-z)_+ \equiv \int dz\, (f(z)-f(1))/(1-z)$ and the color factor $C_F = 4/3$ for $SU(3)$. With the introduction of the scale in $f_1(x;\mu^2)$ one sees that the scale dependence satisfies

$$\frac{\partial f_1(x;\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y}\, P_{qq}\left(\frac{x}{y}\right)\, f_1(y;\mu^2). \tag{109}$$

This is the standard [1] nonsinglet evolution equation for the valence quark distribution function. For the flavor singlet combination of quark distributions or the sea distributions one also needs to take into account contributions as shown in Fig. 6 (right) involving the gluon distribution functions related to matrix elements with gluon fields $F_{\mu\nu}(\xi)$ but otherwise proceeding along analogous lines. The δ-function contribution can be explicitly calculated by including vertex corrections (socalled virtual diagrams), but it is easier to derive them by requiring that the sum rules for f_1 remain valid under evolution, which requires that $\int_0^1 dz\ P_{qq}(z) = 0$.

Except for logarithmic contributions also finite α_s contributions show up in deep inelastic scattering [1]. For instance in inclusive scattering one finds that the lowest order result for F_L is of this type,

$$\begin{aligned} F_L(x_B, Q^2) \quad &= \quad \frac{\alpha_s(Q^2)}{4\pi}\Bigg[C_F \int_{x_B}^1 \frac{dy}{y}\, \left(\frac{2x_B}{y}\right)^2 y\, f_1(y;Q^2) \\ &\quad + \left(2\sum_q e_q^2\right) \int_{x_B}^1 \frac{dy}{y}\, \left(\frac{2x_B}{y}\right)^2 \left(1-\frac{x_B}{y}\right)\, y\, G(y;Q^2)\Bigg], \end{aligned} \tag{110}$$

the second term involving the gluon distribution function $G(x)$.

11. Concluding remarks

In these lectures I have discussed aspects of hard scattering processes, in particular inclusive and 1-particle inclusive lepton-hadron scattering. The goal is the study of the quark and gluon structure of hadrons. For example, by considering polarized targets or particle production one can measure spin and azimuthal asymmetries and use them to obtain information on specific correlations between spin and momenta of the partons. The reason why this is a promising route is the existence of a field theoretical framework that allows a clean study of the observables as well-defined hadronic matrix elements.

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